An enhanced dynamic model for the actuator/amplifier pair in AMB systems

Eric H. Maslen and Carl R. Knospe
Department of Mechanical and Aerospace Engineering
University of Virginia
Charlottesville, VA 22904-4746 USA

Lei Zhu
Calnetix, Inc.
12880 Moore St.
Cerritos, CA 90703 USA

Abstract—Typical models for AMB actuators consist of a static linearized mapping from displacement and current to force \( f \approx -K_x x + K_i i \) and are augmented with actuator dynamics (bandwidth and gain) which act to correlate a command signal \( V_c \) to resulting current: \( I(s) = G_a(s)V_c(s) \).

In the present work, we show that this model misses a number of potentially important features, most notably the effect of amplifier bandwidth limitations on the term mapping displacement to force \( K_x \) and the related influence of journal motion induced back-EMF. In addition, we show how actuator eddy currents interact with the amplifier dynamics and how to model the resulting composite system.

I. INTRODUCTION
Since the beginning of published literature on active magnetic bearings, the standard model for the actuator has relied on a static linearized mapping from displacement and current to force \( f \approx -K_x x + K_i i \) [1]. Later developments included amplifier dynamics in order to achieve higher fidelity. These dynamics were applied in simple way to the existing model: it was assumed that the mapping from amplifier command signal to actual current is characterized by the SISO transfer function \( I(s) = G_a(s)V_c(s) \) so that \( F(s) = G_a(s)K_x V_c(s) - K_x X(s) \) [2], [3].

In the present work, we show that this model misses a number of potentially important features, most notably the effect of amplifier bandwidth limitations on the term mapping displacement to force \( K_x \) and the related influence of journal motion induced back-EMF. In addition, we show how actuator eddy currents interact with the amplifier dynamics and how to model the resulting composite system.

The primary product of the paper is a reformulation so that

\[
F \approx K_x G_a(s) V_a - K_x G_x(s) X
\]

in which \( K_x \) and \( K_x \) are defined in the usual fashion and \( G_a \) is the transfer function of the power amplifier as measured with constant gap. \( G_x(s) \) is similar to \( G_a \) but embeds the transfer function

\[
G_e = \frac{R_0}{R_0 + c \omega}
\]

The parameters \( R_0 \) and \( c \) are readily computed from actuator geometry and material properties.

Several examples are presented to illustrate the differences in predicted behavior between the static linearized model and this newer model, both with and without eddy current effects. In particular, we demonstrate that very stiff suspensions will tend to mask these differences but that soft suspensions — especially those employed when the AMB is used as a test actuator — will tend to emphasize these differences, resulting in very substantial error with the simpler model.

II. DYNAMICS WITHOUT EDdy CURRENTS
Consider a simple single axis AMB actuator with two opposed horseshoe magnets as depicted in Figure 1. Each magnet has \( N \) turns (two coils wired in series, each with \( N/2 \) turns), pole cross section area \( A_p \), and has its pole inclined at an angle \( \theta \) to the vertical direction. The resistance of each series connected pair of coils is \( R_0 \). For such a pair of magnets,

\[
F = \frac{\tau A_p}{\mu_0} (B_1^2 - B_2^2)
\]
and (neglecting eddy current effects)

\[ B_i = \frac{\mu_0 N}{2(90^2 + (-1)^i\gamma x^2)} I_i \] (2)

Further, the coil voltages are related to \( B \) and \( I \) through

\[ V_i = N A_g \frac{dB_i}{dt} + I_i R \] (3)

Now, assume that each of the drive amplifiers has the control law

\[ V_i = G_V(s) V_{ref,i} - G_I(s) R_0 I_i \] (4)

in which \( R_0 \approx R \) and it may reasonably be assumed that \( G_V = \alpha G_I R_0 \) (the constant \( \alpha \) sets the DC transconductance gain) although either may contain some additional filtering aimed at specific closed loop performance.

Define

\[ B_b \equiv (B_1 + B_2)/2 \] (5a)
\[ B_p \equiv (B_1 - B_2)/2 \] (5b)

so that \( B_1 = B_b + B_p \) and \( B_2 = B_b - B_p \) and assume that \( B_b \) is held constant. Rearrange (2) as

\[ I_i = \frac{2(90^2 + (-1)^i\gamma x^2)}{\mu_0 N} B_i \approx \frac{290}{\mu_0 N} B_i + (-1)^i \frac{2\gamma B_b}{\mu_0 N} x \] (6)

With this approximation, we can expand (3) as

\[ V_i = N A_g \frac{df}{dt} + \frac{290 R}{\mu_0 N} B_i + (-1)^i \frac{2\gamma B_b R}{\mu_0 N} x \] (7)

and (4) as

\[ V_i = G_V V_{ref,i} + \frac{290 R}{\mu_0 N} \left( B_i + (-1)^i \frac{\gamma B_b x}{90} \right) \] (8)

Equating (7) and (8) produces

\[ G_V V_{ref,i} = \frac{290 R}{\mu_0 N} \left( B_i + (-1)^i \frac{\gamma B_b x}{90} \right) \]

Define

\[ B_b = \frac{\mu_0 N G_V(0)}{290 R_0 (G_I(0) + R/R_0)} \] (10)

in which it is assumed that \( B_b \) and \( V_b \) are constant so that \( sB_b = 0 \).

Further, define

\[ V_c \equiv \frac{V_{ref,1} - V_{ref,2}}{2} \]

and subtract the \( i = 2 \) instance of (9) from the \( i = 1 \) instance to obtain

\[ B_p = \frac{\mu_0 N G_V}{\mu_0 N^2 A_g + 290 R_0 (G_I + R/R_0)} V_c + \frac{2\gamma B_b R_0 (G_I + R/R_0)}{\mu_0 N^2 A_g + 290 R_0 (G_I + R/R_0)} x \] (11)

This may be written succinctly as

\[ B_p = \frac{\mu_0 N G_A(s) V_c + \gamma B_b G_A(s) x}{290} \] (12)

in which

\[ L_0 \equiv \frac{\mu_0 N^2 A_g}{290} \]
\[ G_A(s) = \frac{G_V(s)}{L_0 s + R_0 G_I(s) + R} \] (13)
\[ G_B(s) = \frac{R_0 (G_I(s) + R/R_0)}{L_0 s + R_0 G_I(s) + R} \] (14)

Now, changing \( B \) coordinates as prescribed by (5), (1)

produces

\[ F = \frac{\gamma A_g}{\mu_0} B_1^2 - B_2^2 = \frac{4\gamma A_g}{\mu_0} B_b B_p \] (16)

Substitute (12) into (16) to obtain

\[ F = \frac{2\gamma A_g B_b N}{90} \left( G_A(s) V_c + \frac{\gamma B_b}{\mu_0 N^2} G_B(s) x \right) \] (17)

Define the control current, \( i \), as

\[ i = \frac{I_1 - I_2}{2} \]

so that

\[ i = G_a \left( V_c - \frac{\gamma B_b N A_g}{290} x \right) \] (19)

The first term, \( G_A(s) V_c \), is the term normally included in magnetic bearing models. The second term depends on the derivative of \( x \): it represents the effect of back-EMF induced by rotor motion and is typically not included in magnetic bearing models.

For comparison to (17), solve (19) for \( V_c \) to obtain

\[ V_c = \frac{1}{G_a(s)} - \frac{\gamma B_b N A_g}{290} G_V(s) x \] (20)

and combine (20) and (17) with definitions (14) and (15) to produce (after some algebra),

\[ F = \frac{2\gamma A_g B_b N}{90} i + \frac{A_g \gamma^2 B_b^2}{\mu_0} x \] (21)

Obviously, (21) is just the usual Taylor's series expansion of (1) in terms of the perturbation current, \( i \equiv (I_1 - I_2)/2 \), and the rotor displacement, \( x \). Thus, assuming the form

\[ F = K_i i - K_x x \]
TABLE I
EXAMPLE THRUST BEARING PARAMETERS
\[
\begin{array}{|c|c|}
\hline
\text{inner radius, inner pole piece} & 51.69 \text{ mm} \\
\text{outer radius, inner pole piece} & 68.32 \text{ mm} \\
\text{inner radius, outer pole piece} & 90.83 \text{ mm} \\
\text{outer radius, outer pole piece} & 101.2 \text{ mm} \\
\text{nominal air gap, } g_0 & 1.0 \text{ mm} \\
\text{bias current, } i_b & 7 \text{ amps} \\
\text{coil turns, } N & 145 \\
\text{coil resistance, } R & 0.232 \Omega \\
\hline
\end{array}
\]

we obtain
\[
K_i = \frac{2\gamma A_g B_b N}{g_0}
\]
and
\[
K_x = -\frac{4A_g \gamma^2 B_b^2}{\mu_g g_0}
\]
so that (17) becomes
\[
F = K_i G_a(s) V_e - K_x G_a(s) x \quad (22)
\]

What is important about (22) is that, including journal motion induced back-EMF terms as in (19) and amplifier dynamics as in (4) modifies both the actuator gain term (coefficient of \(i\) in (21)) and the open-loop stiffness term (coefficient of \(x\) in (21)) imposing bandwidth limits to both terms. Further, by making \(G_V\) and \(G_I\) different, it becomes possible to maintain a high bandwidth product \(K_i G_a\) while reducing the bandwidth of \(K_x G_x\) substantially: this is the essential approach in flux feedback amplifiers. The penalty is that \(G_a\) becomes sensitive at lower frequencies to \(R\), which can vary substantially with temperature.

A. Example: thrust bearing
To examine the effect of this dynamic limiting of \(K_x\), consider a thrust bearing with the dimensions and parameters indicated in Table I. The amplifier transfer function \(G_I = \frac{206x+378}{0.332}\) while \(G_V = 1.5 \frac{206x+378}{s}\) to give a DC transconductance of 1.5 amps/volt. Because \(G_I\) is so large (a proportional gain of 887), the difference between \(G_a\) and \(G_x\) is essentially just a matter of gain, as indicated in Figure 2.

Clearly, the bandwidth of the \(K_x\) effect is the same as that of the \(K_i\) effect. This connection arises because the \(G_I\) and \(G_V\) differ only by a constant ratio. To get a notion of when this dynamic is important, consider (19). If the bus voltage driving the amplifier is 160 volts, then we begin to be concerned about amplifier saturation when the back-EMF term approaches this value. That is, when
\[
\frac{\gamma B_b N A_{gs}}{2g_0 G_V} \approx 160
\]
or, more conveniently, when
\[
\left| \frac{x}{g_0} \right| \approx \frac{320 G_V}{\gamma B_b N A_{gs}}
\]
Since \(x < g_0\), we are interested in frequencies where the term to the right is less than 1.0: at lower frequencies, the voltage problem cannot arise because the motions cannot be large enough. For the present case, the frequency beyond which this condition may be met is readily found to be 52 kHz. It is very unlikely that this thrust bearing will move with an amplitude of ±1.0 mm at a frequency as high as 52 kHz!

To examine the implications for stability, assume that the thrust bearing must control a mass of 34 kg and measures this mass with a sensor with sensitivity of 15000 volts/m. A PID controller is introduced with the transfer function
\[
G_c(s) = \frac{0.00096s^2 + 0.8s + 1}{8 \times 10^{-8}s^3 + 0.0064s^2 + s}
\]
whose Bode plot is provided in Figure 3. With this compensator, the plant including bandwidth limiting on the \(K_x\) term is stable with eigenvalues of -5500, -3800, -2493, -1062, -44.1 ± 719j, and -2.05. If the same system is modeled without bandwidth limiting on the \(K_x\) term, the resulting closed loop system is not stable and has eigenvalues of -5543, -3846, -1126, 7.35 ± 696, -2.05.

Not surprisingly, bandwidth limiting the \(K_x\) term improves the stability of the system (hence the interest in flux feedback). For systems of this sort with relatively...
minor influence of this bandwidth limitation, it is probably
conservative to ignore the effect, although it is not costly
to include it.

III. DYNAMICS WITH EDDY CURRENTS

To consider the effect of eddy currents, we appeal to
the developments in [4]. In that work, it is shown that
the relationship between coil current, gap flux, and gap
variation is closely approximated by

\[
\phi_p(s) = \frac{N}{\mathcal{R}_0 + c\sqrt{s}} f_p(s) - \frac{\delta g}{\partial \tau_p \mu_0 A_g \mathcal{R}_0} \frac{1}{\mathcal{R}_0 + c\sqrt{s}} X_p(s)
\]

in which the subscript \( p \) indicates small perturbations about
an equilibrium point. The coefficient \( c \) characterizes the
eddy current production in the material: large \( c \) implies high
bulk conductivity. The constant nominal circuit reluctance,
\( \mathcal{R}_0 \), is defined for our purposes as

\[
\mathcal{R}_0 = \frac{2g_0}{\mu_0 A_g}
\]

Thus, we may rearrange (23) as

\[
B_i(s) = \frac{\mu_0 N I_b}{2g_0} \frac{1}{1 + c\sqrt{s}} \left( \frac{I_b(s)}{I_b} - (-1)^i \frac{\gamma x(s)}{g_0} \right)
\]

in which \( c' = \frac{\mu_0 A_g}{2g_0} c \)

Following the previous development but using
this modified relationship between flux, current, and
displacement, it is fairly direct to obtain the modified model

\[
F = K_i G_a(s; c') V_e - K_a G_a(s; c') x
\]

\[
G_a(s; c') = \frac{G_v}{L_0 s + (1 + c\sqrt{s}) (G_f R_0 + R)}
\]

\[
G_a(s; c') = \frac{G_v}{L_0 s + (1 + c\sqrt{s}) (G_f R_0 + R)}
\]

A. Example

To illustrate the alteration this produces, consider the
previous example thrust bearing. Here, the parameter \( c \)
has been estimated from experimental work as \( c \approx 0.015 \text{A/m}^2 \). For comparison, the gain and phase
of the transfer functions \( G_a(s) \) and \( G_a(s; c') \) are plotted
in Figure 4. The functions \( G_a(s) \) and \( G_a(s; c') \) have the
same character.

It is particularly interesting to note that both the non–
eddy current model and that including the eddy currents
reach a phase of \(-45^\circ\) at the same frequency (about 400
Hz).

IV. EXTENSION TO GENERALIZED ACTUATORS

For a general actuator, the relationship between the
actuator gap flux density distribution \( \mathcal{B} \) and force in any
given direction \( \gamma \) is [5]

\[
F = F \cdot \gamma = \frac{1}{2\mu_0} B^T A_\gamma B
\]

in which \( A_\gamma = \text{diag}[A_{\gamma i}] \)

and

\[
A_{\gamma i} = -A_i \cdot \gamma
\]

that is, the diagonal elements of \( A_\gamma \) are the dot product of
the outward normal to each pole's area and the direction
\( \gamma \) in which the force is measured. The magnitude of \( A_i \)
is the gap area of the \( i^{th} \) pole face.

The vector of gap fluxes \( \mathcal{B} \) is related to the coil currents
\( I \) and rotor position \( \varphi \) by

\[
\mathcal{R}(\varphi) \mathcal{B} = \mathcal{N} I
\]

By Faraday's and Ohm's laws, the voltages across the coils are

\[
V = \mathcal{N} \frac{d}{dt} \mathcal{B} + R I
\]

Differentiating (29) produces

\[
\mathcal{R}(\varphi) \frac{d}{dt} \mathcal{B} + \sum_{i=1}^{n} \frac{d \mathcal{R}_i}{d \varphi_i} \frac{d \varphi_i}{dt} = \mathcal{N} \frac{d I}{dt}
\]

so that, assuming \( \mathcal{R}^{-1} \) exists,

\[
\frac{d}{dt} \mathcal{B} = \mathcal{R}^{-1} \left( \mathcal{N} \frac{d I}{dt} - \sum_{i=1}^{n} \frac{d \mathcal{R}_i}{d \varphi_i} \frac{d \varphi_i}{dt} \right)
\]

and, finally,

\[
V = \mathcal{N} \mathcal{R}^{-1} \left( \mathcal{N} \frac{d I}{dt} - \sum_{i=1}^{n} \frac{d \mathcal{R}_i}{d \varphi_i} \frac{d \varphi_i}{dt} \right) + R I
\]
Typically, the coils are wound in series sets so that
\[ V_s = C^T V \]
which dictates that the coil currents in the coils of these series sets are given by
\[ I = C L_s \]
so that the coil voltages for the series sets are governed by
\[ V_s = C^T N^R^{-1} \left( \sum_{i=1}^{n} \frac{d^2}{dt^2} \frac{dR}{dx_i} + C^T R C L_s \right) \]
Define
\[ L_s = C^T N^R^{-1} N C \bigg| \xi = \xi_0 \]
\[ Q = [Q_i] \quad Q_i = R^{-1} \frac{dR}{dx_i} \bigg| \xi = \xi_0, B = B_0 \]
and
\[ R_s = C^T R C \]
to obtain the simpler statement
\[ V_s = L_s \frac{dL_s}{dt} - C^T N Q S g \frac{dS}{dt} + R_s L_s \]  
(34)
Now, assume a control law for the amplifier array:
\[ V_s = G_v(s) V_s - R_s, o G_l(s) I_s \]
so that
\[ (L_s + R_s + R_s, o G_l(s)) L_s = G_v(s) V_s + C^T N Q S g \]
or,
\[ L_s = (L_s + R_s + R_s, o G_l(s))^{-1} (G_v(s) V_s + C^T N Q S g) \]
which may be written as
\[ L_s = G_s(s) (V_s + G_v(s) C^T N Q S g) \]  
(35)
in which
\[ G_s(s) = (L_s + R_s + R_s, o G_l(s))^{-1} G_v(s) \]
Referring back to (28),
\[ F_s \approx \frac{1}{\mu_0} B^T A_y \frac{\partial}{\partial \xi} B (L_s - L_s, 0) \]
\[ + \frac{1}{\mu_0} B^T A_y \frac{\partial}{\partial \xi} B (\xi - \xi_0) \]  
(36)
Define
\[ K_{i,v} \equiv \frac{1}{\mu_0} B^T A_y R^{-1} N C \quad \text{and} \quad K_{i,v} \equiv \frac{1}{\mu_0} B^T A_y Q \]
so that
\[ F_s \approx K_{i,v} (L_s - L_s, 0) - K_{i,v} (\xi - \xi_0) \]  
(37)
As previously, substitute (35) into (37) to obtain
\[ F_s \approx K_{i,v} G_s(s) V_s + (K_{i,v} G_v C^T N Q S g - K_{i,v} \xi_0) \xi_0 \]  
(38)

Fig. 5. Schematic view of the actuator/amplifier/rotor interaction. Eddy currents are not indicated

V. SYSTEM OBSERVATIONS

One useful feature of this expanded view of the dynamics of the amplifier/actuator interaction is that it makes accessible some important signals. An obvious way to model the interaction is indicated in Figure 5. With this structure, signals like coil voltage \( V \) and perturbation flux \( \phi \) become accessible as part of the model signal set. These signals can be very useful in evaluating system performance — where voltage should be compared to amplifier supply voltage and flux (plus bias flux) should be compared to saturation levels for the actuator. This is especially valuable when synthesizing controllers using methods like \( H_\infty \) or \( \mu \) where cost functions should explicitly weight these signals.

VI. CONCLUSIONS

The dynamic interaction of the actuator, amplifier, and rotor motion of an active magnetic bearing were reformulated to properly account for the effects of finite amplifier bandwidth on not only the actuator's effective gain but also its negative stiffness. Although the effect for practical bearings is not strong, the formulation does offer higher fidelity than existing models and has advantages in terms of available signals when used in some control synthesis frameworks. Further, the effect of eddy currents on these properties was also explored using a simple fractional derivative model which has been shown to exhibit high fidelity. Extension to generalized actuators was developed leading to a matrix formulation with a form similar to the foregoing scalar result.

REFERENCES